

HW 5 Duality

m 5500 Calculus of variations, Spring 2015

1. Find the Legendre transform $f^*(x^*)$ of $f(x) = e^{3x}$. Plot both functions.
2. Find f^* and f^{**} if

$$f(x) = \frac{1}{2} \min \left\{ (x-1)^2, (x+1)^2 \right\}$$

Draw the graphs.

3. The energy W is

$$W = \frac{\sigma}{2} e^T e + e^T a + c, \quad e = \nabla u$$

where $\sigma > 0$ and c are constants, a is a constant vector.

Find The dual energy $W^*(e^*)$ and show that $\nabla \cdot e^* = 0$

Show that the work $A(e, e^*) = e^T e^* = W(e) + W^*(e^*)$ is independent of c .

Find minimal possible work $\min_{e, e^*} A(e, e^*)$ and the minimizers

$$(e_0, e_0^*) = \arg \min_{e, e^*} A(e, e^*)$$

.