

## HW 4 Multivariable Euler-Lagrange equation

m 5500 Calculus of variations, Spring 2015

1. The thermal equilibrium is described by the boundary value problem for the temperature  $T(x)$

$$\delta T = \gamma(x)T \text{ in } \Omega, \quad \frac{\partial T}{\partial n} + \alpha T^4 = T_0 \text{ on } \partial\Omega,$$

where  $\gamma$  is the density of heat sources,  $\alpha$  is the radiation constant,  $T_0$  is a constant outside temperature.

Write the variational problem which minimizer describes the equilibrium.

2. Find the Euler-Lagrange equation:

$$I = \min_{u:u|_{\partial\Omega}=u_0} \int_{\Omega} |\nabla u|^p dx, \quad \geq 1$$

Consider the case when  $p \rightarrow \infty$ .