

## HW 4, 5500-2019

- Find Hamiltonian and the dual problem for the brachistochrone.
  - Find the upper and lower bounds by choosing some trial functions instead of minimizers.
- Find the conjugate to  $L(z) = -\log(z)$
- The energy  $V$  in an equilibrium is defined up to a constant  $C$

$$V(u', u) = \frac{\alpha}{2}(u')^2 + \beta u' + \frac{\gamma}{2}u^2 + C$$

- Find the dual with respect to  $u'$  energy  $V^*(p, u)$ .
- Show that the sum  $V + V^*$  does not depend on  $C$ .
- What quantity does this sum estimate?