5500-18 HW 2

1. **"Inverse brachistochrone":** Assume that the fastest trajectory between points $A = (1, 0)$ and B=$(0, 1)$ is a part of a circle

$$y(x) = \sqrt{1 - x^2}$$

Find the dependence of speed $v(y)$ on y.

2. Using transversality condition and brachistochrone equation, find the fastest path from the point $(0, 0)$ to a point on the circle $x^2 + y^2 = 1$.

3. Consider the 1-periodic "square wave"

$$f(x) = 2\left[H(x) - H\left(x - \frac{1}{2}\right)\right] - 1 \quad x \in [0, 1], \quad f(x + 1) = f(x)$$

where $H(x)$ is Heaviside function. The Fourier series of f is

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,..}^{\infty} \frac{1}{n} \sin(2n\pi x).$$

1. Find the smooth approximation of f, writing the variational problem

$$\min_{u(x)} \int_0^L \left[p + (f - u)^2\right] dx$$

where the stabilizer p is:

a) $p = p_a = \alpha^2 (u')^2$

b) $p = p_b = \alpha^2 (u'')^2$

2. Find the Fourier series for the approximations u_a and u_b.

3. Graph $f(x)$ and the approximations u_a and u_b using different values of α.