HW 5

1. Analyze the problem

$$I = \inf_{u(x)} \int_0^2 F(x, u, u')dx, \quad x(0) = 0, x(2) = 2$$

with nonconvex Lagrangian

$$F(x, u, u') = (u - x)^2 + G(u'), \quad G(z) = \min\{z^2,\ (z - 2)^2 + 1\}$$

Find the region of non-convexity, relaxed Lagrangian (convex envelope), oscillating solution.

2. Formulate control problem for optimal fuel consumption of the car that moves from $x = A$ to $x = B$ along the hilly road. The goal is to drive from A to B in a given time T, minimizing the total fuel consumption. Assume that the forces acting on the car are:

- gravity component $g(x)$ (due to uneven terrain), the known function,

- viscosity $V(\dot{x}) = -\gamma\dot{x}$, where $\dot{x}$ is the speed and γ is a constant,

- inertia $m\ddot{x}$, where m is the mass

- motor force f, $f(u) = \beta\sqrt(u)$ where u is the rate of fuel consumption, $0 \leq u \leq C$ and C is a positive constant

Formulate the control problem, write the optimality conditions. Analyze the obtained ODEs.