HW3: Constrained Problems

1. **[Lagrange Multiplyers. Duality]**

   A. Find the minimal distance between point $(1, 0, -1)$ and the plane $2x - y + z = 5$. Formulate the dual problem and solve it.

   B. Find the minimal distance between point $(1, 0, -1)$ and the line $x + y = 3$, $x - z = 2$. Formulate the dual problem and solve it.

   What is the relation between these problems?

2. **[Isoperimetric problem. Inequality constraint]**

   A heavy chain of the length $L = 4$ is hanged over a floor $h = 0$, a part of the chain lies on a floor. The coordinates of the supports are $h = 1,\ x = 0$ and $h = 1, x = 1$. Find the equilibrium position of the chain.

   Hint: Find the equation for the point where the chain touches floor using the length and shape of the chain. The problem of the heavy chain is described in the note "Constrained problems."

3. **[Inverse Problem]** The thermal equilibrium in a bar $0 \leq x \leq 1$ heated by the thermal source $\gamma(x)$ is described by the boundary value problem for the temperature $T(x)$

   $$T'' = \gamma(x) \in (0 \leq x \leq 1), \quad T' + \alpha(T - T_0)^4 = 0 \text{ at } x = 1,\ T' = 0 \text{ at } x = 1.$$

   where γ is the density of heat sources, α is the radiation constant, T_0 is a constant outside temperature.

   Write the variational problem which Euler equation describes the equilibrium.