HW1

1. Derive the Euler equation. Solve

$$\min_{u(x)} \int_0^b \left((u')^2 + 2x\,u\right) dx \quad u(1) = 0, \quad u(b) = 0,$$

2. Derive the Euler equation and transversality condition. Solve

$$\min_{u(x),b} \int_0^b \left((u')^2 - 2\,u + x\right) dx \quad u(0) = 1, \quad u(b) = 0,$$

3. Derive the Euler equation and boundary conditions. Solve. Find the first integral

$$\min_{u(x)} \left[\int_0^1 \left((u')^2 - \omega^2 u^2\right) dx + 2u(1),\right] \quad u(0) = 0,$$

4. Derive the Euler equation and boundary conditions. Solve. Explain the dependence of the solution on a.

$$\min_{u(x)} \int_0^b \left((u')^2 + a\,u\,u' + u\right) dx \quad u(0) = 0.$$

5. Derive the Euler equation and boundary conditions. Solve, using Maple

$$\min_{u(x)} \int_0^b \left((u'')^2 - a\,(u')^2 + b\,u\right) dx \quad u(0) = 0, \quad u'(0) = 0$$

6. Assume that a planet of viscous gas permits for the speed $v = c\,r$ where r is the distance from the center. Describe the brachistochrone: the fastest path between the given points $(R, 0)$ and (R, Θ) in polar coordinates (r, θ). (We assume that the path is a curve in the optimal plane that passes through points of origin and destination, and the center of the planet.)