History of Mathematics. Math 3010-001. Fall 2023 Syllabus and Policies

Dates and time: Mo We Fr 11:50 am-12:40 pm.
Location: LCB 225
Final exam: Thursday, December 14, 2023 10:30 am - 12:30 pm
Instructor: Andrej Cherkaev
Email: cherk@math.utah.edu
Office hours: JWB 225. Monday 1-2 pm and by appointment

Text: Victor J. Katz. A History of Mathematics (3rd Edition), Pearson 2009.

Description

Theme Mathematics has been a hallmark of every society since recorded history began. Its complexity reflects a civilization's ability to handle logic, quantities, shapes, processes, arrangements, etc. Today, mathematics surrounds us in computers, engineering, architecture, medicine, transportation, art, money, and sports.

The course offers a journey through the four millennia of mathematical research. We track the history of algebra, geometry, discrete mathematics, calculus, and statistics. We look at ancient societies' contributions and recent developments observing the graduate rise of sophistication and level of abstraction of concepts, and learn about great mathematicians and their ideas. We review some central problems of the times and practice their solutions.

The expected learning outcomes At the end of the course, the students should understand the tendencies and social reasons for the development of mathematics, the methods of ancient mathematicians, and the ideas behind modern mathematical branches. The students should learn landmark problems that lead to breakthroughs in mathematical practices; learn about the works of great mathematicians in the past and present. Finally, students should polish their skills in writing essays.

The course M3010 satisfies the General Education requirements; it addresses the following Essential Learning Outcomes: Critical Thinking, Written Communication, Foundations, and Skills for Lifelong Learning.

Course Design The course consists of lectures, discussions, homework, essay writing, peer reviews, and consultations with the instructor.

The students will read the assigned material before class and be ready for the discussion. A part of the grade assesses the in-class activity (questions and discussion).

Assignments, Assessment, and Grading The homework will be assigned every week or two. The midterm and final essays serve as midterm and final exams.

The grade is based on the following:
- Homework problems - 35%.
- Participation in class discussions and presentations - 10%.
- Peer reviews - 10%
- Introductory essay - 10 %
- Midterm Essay on Math before 19th century - 15%
- Final Essay on development of a mathematical concept - 20%.

Policies

Checking grades Please check the accuracy of the homework, online assignments, and exam grades. Keep a record of all your graded assignments. Contact the instructor quickly if you see any error in Canvas's grades record.

Late Assignments and Regrading I will be flexible where possible/ appropriate. Please remember, however, that grading late assignments is a pain. You may ask for regrading provided you have a good reason.

Incompletes According to university policy, to be considered Incomplete, a student must have 20% or less of the course work remaining and be passing the course with a C or better. You must request an incomplete grade, and I will consider giving that grade under appropriate circumstances.

Content Accommodations Consistent with principles of academic freedom, the faculty, individually and collectively, is responsible for determining the content of the curriculum. Students are expected to take courses that will challenge them intellectually and personally. Students must understand and articulate the ideas and theories important to the discourse within and among academic disciplines. A personal disagreement with these ideas and

theories or their implications is not sufficient grounds for requesting the accommodation (see
https://regulations.utah.edu/academics/6-100.php).

Plagiarism Plagiarism is unacceptable and results in **F** - grade. I use a plagiarism detection service, and I test the ChatGPT involvement.

Syllabus topics

Intro Branches of contemporary math. Math, art, and natural sciences.

Anchient Mathematics

Prehistory: math originated in the cradles of civilization: Sumer-Babylon, Egypt, The Indus Valley, China, England, etc.

Bronze Age (3000- 1000 BC): Almost all unearthed documents come from Sumer-Babylon and Egypt. Base 60 numerals, multiplication, division, a geometrical solution of quadratic equations, Pythagorean triples, and the iterative algorithms for square roots. Egyptian-Roman numerals, approximation of Pi, binary system, Egyptian fractions.

Greek Math: Pythagoras and this society, Zeno and Aristoteles logic, Archimedes and the beginning of calculus, Euclid - geometry and number theory, Ptolemy - a model of the Solar system.

In between: Outlook of Chinese, Indian, Islamic, and medieval European math: Numeral system (base 2 and base 10), negative numbers, the story of zero, trigonometry, algorithms, etc.

16th century: Discovery of logarithms (Napier). Solution of the cubic equations and introduction of complex numbers (Tartaglia, Cardano, Bombelli). Works of Copernicus, Galileo, and Kepler.

Students will write essays on ancient math. The paper should have a story suitable for middle or high school students, and the articles will be collected and published.

Foundations of Modern Mathematics

The 17th-century mathematics during **Scientific Revolution:** Works of Newton, Leibniz, Fermat, Pascal, Descartes, Huygens, *at al*: Introducing calculus, functions, coordinates, differential equations, probability.

The 18th-century mathematics during **Age of Reason**: Bernoulli family, Euler, Lagrange, Laplace, Monge. Introducing Calculus of variations, series, algebraic notations, modeling of nature, topology, complex-valued functions, and gravity law.

Modern Mathematics

19th-century mathematics during **Industrial revolution**

Development of analysis, Fourier series, stability, complex analysis, divergent series, differential equations. (Lagrange, Laplace, Gauss, Sophy Germen, Cauchi, Abel, Galois)

Vectors and matrices - Agrand, Gauss, Hamilton, Cayley, Sylvester, quaternions: Maxwell.

Formalization of gravity, heat transfer, elasticity, electromagnetism, and fluid dynamics.

Non-Euclidean geometry, curved spaces, differential geometry (Gauss, Lobachevsky, Bolyai, Riemann).

Algebra: Solvability of equations, group theory. Boolean logic. (Lagrange, Galois, Lie, Boole)

Probability and statistics (Legendre, Gauss, Laplace, Chevyshev, Pierson)

Generalization of functions, set theory, paradoxes (Weierstrass monster function, Cantor)

Faces and dramas behind theorems: Lagrange, Laplace, Sophie Germain, Fourier, Cauchy, Abel, Galois, Gauss, Lobachevsky, Riemann, Weierstrass, Chebyshev, Agrand, Hamilton, Moebius, Maxwell, Cayley, Sylvester, Heaviside, Kelvin

Women in modern mathematics Maria Agnesi, Sophie Germain, Sofya Kovalevskaya, Emmy Noether, Olga Ladyzhenskaya, Maryam Mirzakhani, Karen Uhlenbeck, and others.

Glance into contemporary mathematics (20-21 centuries)

Possible topics:

What are the pure and applied mathematics?

Foundations: from Cantor to Continuum hypothesis, Zermelo-Fraenkel Axioms, and Goedel theorem.

Analysis: Hilbert space, distributions, measures, and integration. Aggregation: chaos and homogenization.

Diff. Equations: Nonlinear phenomena, bifurcations, singularities, blow-up solutions, etc.

Geometry: Fractals, aperiodic tiling, graphs. Computer-aided proofs.

Numerical methods. Math modeling. Optimization, game theory, engineering math, financial math, transport problem, and metamaterials.

Big Data. Machine learning. Computer-aided proofs.

Students will write essays on the topic of contemporary math. The paper should highlight the history of the concept, its importance, the specific contributions of the researchers, and their bio-sketches.

Table 1: 3010. Preliminary weekly schedule - Fall 2023

Week	Subject	Text
	Antique Mathematics	
1	Introduction. Structure and roots of modern math. Prehistoric math, Numeral system	Notes
2	Mathematics in Bronze Age: 2nd millennium BCE. Mesopotamia, Egypt	Ch. 1
3 -4	Greek math: Pythagoras, Archimedes, Euclid.	Ch 2-3
4 - 5	Math in China, India, Mediavistic Muslim world and medieval Europe.	Review Ch 7-11
6	Fall break	
	Origins of Modern math. 16-18th centuries	
7	Cubic equation and complex numbers. Logarithms. Works of Fermat, Pascal, Descartes: Number theory, Probability, Analytic geometry	Ch.12-14
8	Newton and Leibniz: Function, Calculus and Differential Equations, first computer	Ch. 16
9	Bernoulli, Euler: Calculus of Variations, Series, Beginning of Topology	Ch.16-17
	Branches of modern math. 19th century	
10	Complex Analysis, Differential equations. Fourier series	Ch. 18
11	Algebra: groups, vectors, matrices, Probability	Ch. 22
12	Differential and non-Euclidean geometry	Ch. 24
	Glance through 20th-21st century math	
13	Continuum hypothesis, Goedel theorems, Fermat theorem, aperiodic tiling, Kepler's conjecture,	Ch. 25,
14	Limits: Hilbert space, Distributions, Measures in PDEs, Fractals, Chaos, Homogenization.	Ch. 25
15	Math modeling. Optimization, Game theory, Transport problem, Metamaterials.	Internet
16	Probability and Statistics. Big Data. Numerics, Machine learning, and computer-aided proofs.	Internet.

The schedule may be changed This syllabus is meant to serve as an outline and guide for our course. I may modify the plan to respond to the needs of our class.

Essays: Important dates

September 6. Submit the introductory free-style essay
"Mathematics in our world."

October 11. Submit the project of the midterm essay for approval.
October 18. Submit the draft of the essay for peer review.
October 25. Submit the peer reviews.
November 1. Submit the midterm essay.

November 15. Submit the project of the final essay for approval.
November 22. Submit the draft of the essay for peer review.
November 29. Submit the peer reviews.
December 8. Submit the final essay.