Math 6510 - Homework 10

Due at 4 PM on 12/3/04

1. Prove the Jacobi identity:

$$[[v_1, v_2], v_3] + [[v_2, v_3], v_1] + [[v_3, v_1], v_2] = 0$$

2. Let $v_1, \ldots, v_k$ be vector fields defined in a neighborhood of 0 in $\mathbb{R}^n$. Let ϕ_t^i be the respective flows. Define

$$\chi(x_1, \ldots, x_n) = \phi^1_{x_1} \circ \phi^2_{x_2} \circ \cdots \circ \phi^k_{x_k}(0, \ldots, 0, x_{k+1}, \ldots, x_n)$$

   If $v_1(0), \ldots, v_k(0)$ are linearly independent show that $d\chi_0$ is an isomorphism and that $\chi_*(\frac{\partial}{\partial x_1}) = v_1$.