Math 6510 - Homework 1

Due at 4 PM on 9/15/04

1. Find an differentiable atlas for $S^1 \times S^1$,
2. Show that if M and N are differentiable manifolds then $M \times N$ is a differentiable manifold.
3. Find a differentiable atlas for $\mathbb{R}$ such that the identity map is not smooth.
4. Show that for every differentiabl structure on $\mathbb{R}$ there is a smooth, strictly increasing function from $\mathbb{R}$ to $\mathbb{R}$ where the second copy of $\mathbb{R}$ has the standard structure. Use this to show that any two differentiable structures on $\mathbb{R}$ are diffeomorphic.
5. $SL_n(\mathbb{R})$ is the space of $n \times n$ matrices with determinant one. Show that $SL_n(\mathbb{R})$ is a differentiable manifold of dimension $n^2 - 1$. (Hint: The space of all $n \times n$ matrices is naturally homeomorphic to $\mathbb{R}^{n^2}$. The determinant is then a map from $\mathbb{R}^{n^2}$ to $\mathbb{R}$. Show that 1 is regular value of this map.)
6. Do #5,#7 in Section 4 of Guillemin and Pollack (pages 25-26).