SOLUTIONS

University of Utah
Department of Mathematics
1010-1 Intermediate Algebra - Spring 2006
Instructor: Marian Bocea

Monday, April 24, 2006

NAME (please print clearly):

Midterm # 4

Question 1 (20 points):

Question 2 (20 points):

Question 3 (20 points):

Question 4 (20 points):

Question 5 (20 points):

TOTAL SCORE:

Notes:
1. You have 50 minutes to complete the test.
2. For full credit you must show your work completely. You need to clearly indicate which formulas or theorems you are using in the process of solving the problems. Simply writing down an answer without justifying it will receive very little partial credit.
3. NO TEXTBOOKS, NOTES or CALCULATORS are allowed while you take the test.

1. (a) (10 points) Find the domain of the function $f(x) = \sqrt[4]{2x-4}$, and compute $f(10)$.

(b) (10 points) Use the laws of exponents to simplify the expression

$$\frac{\sqrt[4]{2x+y}}{(2x+y)^{\frac{3}{2}}}$$

Solution: (a) The domain of f is the set

8p. $\{x \in \mathbb{R} \mid 2x-4 \geq 0\} = \{x \in \mathbb{R} \mid x \geq 2\} = [2, +\infty)$

4p. $f(10) = \sqrt[4]{2 \cdot 10 - 4} = \sqrt[4]{16} = \sqrt[4]{2^4} = 2$

(b) We have:

8p. $$\frac{\sqrt[4]{2x+y}}{(2x+y)^{\frac{3}{2}}} = \frac{(2x+y)^{\frac{1}{4}}}{(2x+y)^{\frac{3}{2}}} = (2x+y)^{\frac{1}{4}-\frac{3}{2}} = (2x+y)^{-\frac{5}{4}} = \frac{1}{(2x+y)^{\frac{5}{4}}}$$

2. (20 points) Rationalize the denominator of the expression, and simplify further, if possible.

$$\frac{5x^2y}{\sqrt{5x+y}-\sqrt{y}}$$

Solution:

15p.
$$\frac{5x^2y}{\sqrt{5x+y}-\sqrt{y}} = \frac{5x^2y\left(\sqrt{5x+y}+\sqrt{y}\right)}{\left(\sqrt{5x+y}-\sqrt{y}\right)\left(\sqrt{5x+y}+\sqrt{y}\right)}$$
$$= \frac{5x^2y\left(\sqrt{5x+y}+\sqrt{y}\right)}{5x+y-y}$$
$$= \frac{5x^2y\left(\sqrt{5x+y}+\sqrt{y}\right)}{5x}$$

Thus,

$$\frac{5x^2y}{\sqrt{5x+y}-\sqrt{y}} = xy\left(\sqrt{5x+y}+\sqrt{y}\right)$$ 5p.

3. (20 points) Solve the radical equation

$$\sqrt{x-10} + \sqrt{x+2} = 1$$

Solution: The eqn is equivalent to

$$\sqrt{x+2} = 1 - \sqrt{x-10}$$

5p.

Taking the square, we obtain

$$x+2 = 1 + x - 10 - 2\sqrt{x-10}$$

Equivalently,

$$2\sqrt{x-10} = -11$$

7p

Since $2\sqrt{x-10} \geq 0$ for all values of $x \in \mathbb{R}$ for which the radical is well defined, we deduce that the eqn. has no solutions.

8p.

4. (20 points) Perform the operation, and write the result as a complex number in standard form $a + bi$ $(a, b \in \mathbb{R})$

$$\frac{1}{2+3i} - \frac{2}{1-i}$$

Solution:

8p.

$$\frac{1}{2+3i} - \frac{2}{1-i} = \frac{1-i}{(2+3i)(1-i)} - \frac{2(2+3i)}{(1-i)(2+3i)} =$$

$$= \frac{1-i-4-6i}{2+3i-2i+3} = \frac{-3-7i}{5+i} =$$

6p.

$$= \frac{(-3-7i)(5-i)}{(5+i)(5-i)} = \frac{-15+3i-35i-7}{25+1}$$

6p.

$$= \frac{-22-32i}{26} = -\frac{22}{26} - \frac{32}{26}i =$$

$$= -\frac{11}{13} + \left(-\frac{16}{13}\right)i$$

Alternatively, $\frac{1}{2+3i} - \frac{2}{1-i} = \frac{2-3i}{13} - \frac{2(1+i)}{2} = \frac{2}{13} - 1 - \frac{3}{13}i - i = -\frac{11}{13} + \left(-\frac{16}{13}\right)i$

5. (20 points) Solve the quadratic equation

$$2x^2 + 3x + 5 = 0$$

Solution Using the quadratic formula

5p. the solutions are

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot 5}}{2 \cdot 2}$$

7p.

$$= \frac{-3 \pm \sqrt{9 - 40}}{4}$$

$$= \frac{-3 \pm \sqrt{-31}}{4}$$

8p.

$$= \frac{-3 \pm \sqrt{31}\, i}{4} = -\frac{3}{4} \pm \frac{\sqrt{31}}{4} i$$

There are two complex conjugate solutions:

$$-\frac{3}{4} + \frac{\sqrt{31}}{4} i \quad \text{and} \quad -\frac{3}{4} - \frac{\sqrt{31}}{4} i$$