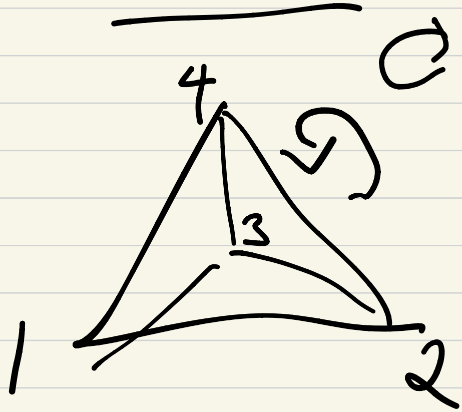
# 4800-8 (Part!)




Tetrahedron

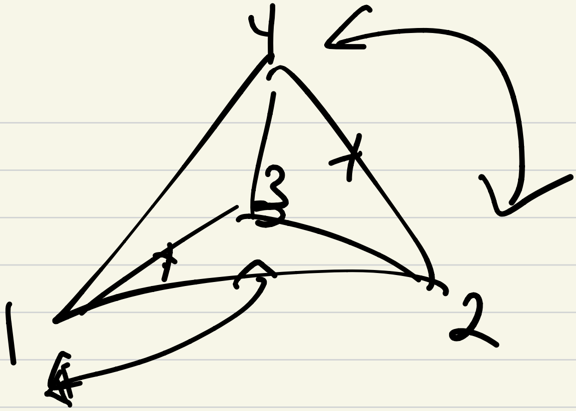
Rotations about a vertex:

(1) (243), (234)

(2) (134), (143)

(3) (124), (142)

(4) (123), (132)




About opposite edges.

$$\left\{ \begin{array}{l} (1\ 3)(2\ 4) \\ (2\ 3)(1\ 4) \\ (1\ 2)(3\ 4) \end{array} \right.$$

$$\boxed{\mathrm{id}}$$

$A_4 = \left\{ \text{even permutations of } [4] \right\}$

All permutations of [4]:

id

$(i\ j)$ one tr $\binom{4}{2}$

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product two transpositions

disjoint or overlap

$(i\ j) \circ (k\ l)$

$= (i\ j)(k\ l)$

edge

$(i\ j) \circ (i\ k)$

$= (i\ k\ j)$

3-cycle

vertex symmetry

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**Remaining:** 4 cycles.

__Rmk:__ Composing two edge symmetries is an edge symmetry.

$$(13)(24) \circ (12)(34)$$

$$= (14)(23)$$

| $\cdot$ | $\overset{a}{(12)(34)}$ | $\overset{b}{(13)(24)}$ | $\overset{c}{(14)(23)}$ |
|---|---|---|---|
| $\overset{a}{(12)(34)}$ | id | c | b |
| $\overset{b}{(13)(24)}$ | c | id | a |
| $\overset{c}{(14)(23)}$ | b | a | id |

$$\{1, (12)(34), (13)(24), (14)(23)\}$$

$$= K_4 = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$$

$$= (\mathbb{Z}/8\mathbb{Z})^{+}$$

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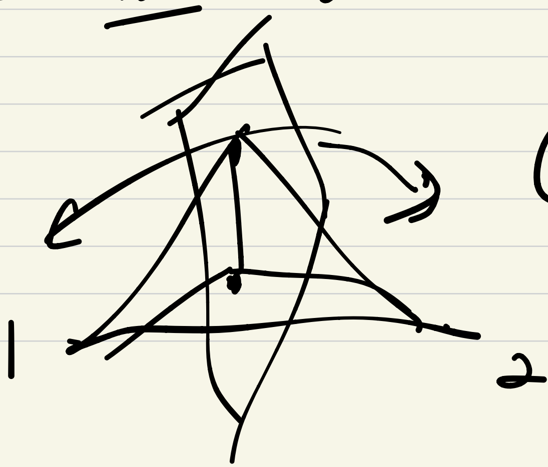
<u>Groups</u> <u>w/</u> <u>small</u> <u>#'s</u>
of <u>element</u>

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| id | $C_2$ | $C_3$ | $C_4$ or $K_4$ | $C_5$ |

$C_6$

$C_6$ or $D_6 = S_3$ (symmetries of [3])

Instead imagine symmetries of  that are not orientation preserving:




$(1\ 2)$

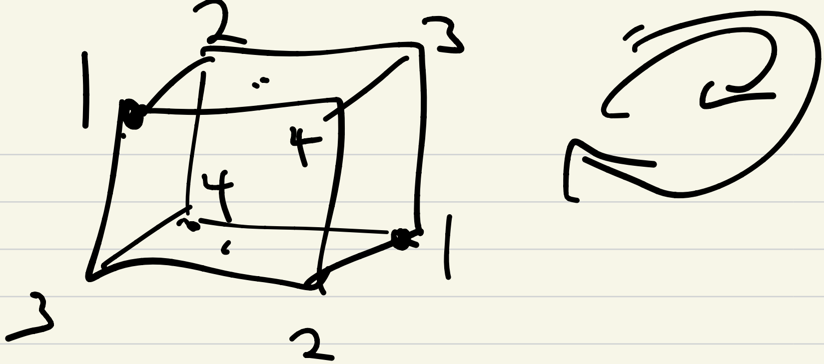
Upshot: All $S_4$

consists of symmetries

of the tetrahedron:

$\to A_4$ $\quad$ $\det = +1$

$\to A_4 \cdot (1\ 2)$ $\quad$ $\det = -1$




<u>Claim</u>: Orientation preserving symmetries $= S_4$

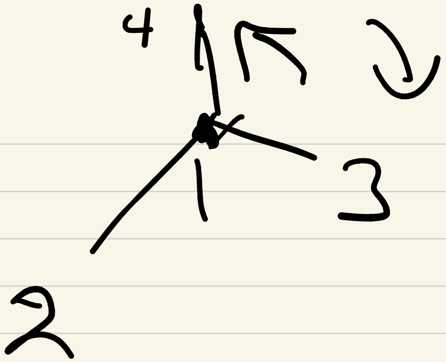
<u>Mark</u> ends of diagonals: $1, 2, 3, 4$.

<u>Rotate about a square</u>: (horizontal)

$(1\ 2\ 3\ 4) = \rho \leftarrow$ odd ✓

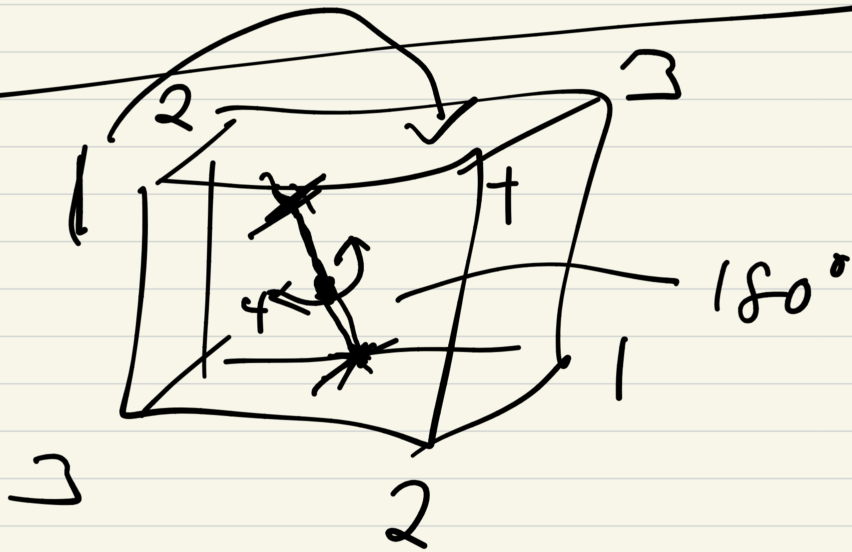
$(1\ 3)(2\ 4) = \rho^2 \leftarrow$ even $\times 3$

$(1\ 4\ 3\ 2) = \rho^3 \leftarrow$ odd




$(2\ 3\ 4)$ $\times\ 4$

$(2\ 4\ 3)$




$(1\ 4)$ . . . transpositions!

2 fixed, 3 fixed !

What happens if you allow the cube to invert?

24

$\mathrm{Sym}(\square)$ orientation preserving

$\rho$

$\mathrm{Sym}(\square) \cdot$ (reflection about origin) 24

$\boxed{\text{Gp w/ 48 elements}}$

# Vector Spaces

**Fields:** $(F, +, \cdot, 0, 1)$

set

$(F, +, 0)$ is an abelian gp

$(F^*, \cdot, 1)$ is an an abelian gp.

|| Multiplication distributes w/ addition. ||

**Examples:** $\mathbb{Q}, \mathbb{R}, \mathbb{C}$

$\mathbb{Z}/p\mathbb{Z}$

$$R(t) = \frac{f(t)}{g(t)}$$

rational fcns. in one variable

$\mathbb{C}(t)$ (complex function)

Alg. Geometers

$\mathbb{C}(t_1, \ldots, t_n) \subseteq K$

finite

Number Theorists

$\mathbb{Q} \subseteq K$ (e.g. $\mathbb{Q}[i]$)

finite

$a+bi$

A field $k$ is alg. closed

if every poly. with coeff. in $k$ has a root in $k$

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E.g. $\mathbb{R}$ is not $x^2+1$

$\mathbb{Q}$ is not $x^2-2$

$\mathbb{Z}/p\mathbb{Z}$ is not.

$x^2-a^2$ in $\mathbb{Z}/p\mathbb{Z}$ $\pm a$

or none

Es. $\mathbb{Z}/7\mathbb{Z}$

$x^2-1$ has roots $1, 6$

$x^2-2$ has roots $3, 4$

$x^2-3$

$x^2-4$ has roots $2, 5$

$x^2-5$

$x^2-6$

no roots.

$\mathbb{C}$ is algebraically closed

Important: Finding eigenvalues & Eigenvectors

A vector space $(V, +, 0)$ over $F$ is an abelian gp with scalar multiplication:

$$\cdot : F \times V \longrightarrow V$$

bilinear & $a(b\vec{v}) = (ab)\vec{v}$.

Examples of vector spaces:

$F^n = F \times \cdots \times F$

with coordinatewise addition

and

$\lambda(a_1, \ldots, a_n) =$

$(\lambda a_1, \ldots, \lambda a_n)$.

These are (up to isomorphism)

all of the finite-dim'l

vector spaces.

Def: An $F$-linear map of $F$-vector spaces.

$$f : V \longrightarrow W \quad \text{is}$$

(a) linear $f(\vec{u}+\vec{v}) = f(\vec{u}, \vec{v})$

(b) $f(a\vec{v}) = a \cdot f(\vec{v})$

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$\mathrm{Vec}_F$ = category of vector spaces $/F$

- $f: F^n \to F^n$

$\checkmark$ $\underline{\det(f)}$

- $\underline{\text{Jordan normal form}}$
  $\underline{\text{of } f.}$