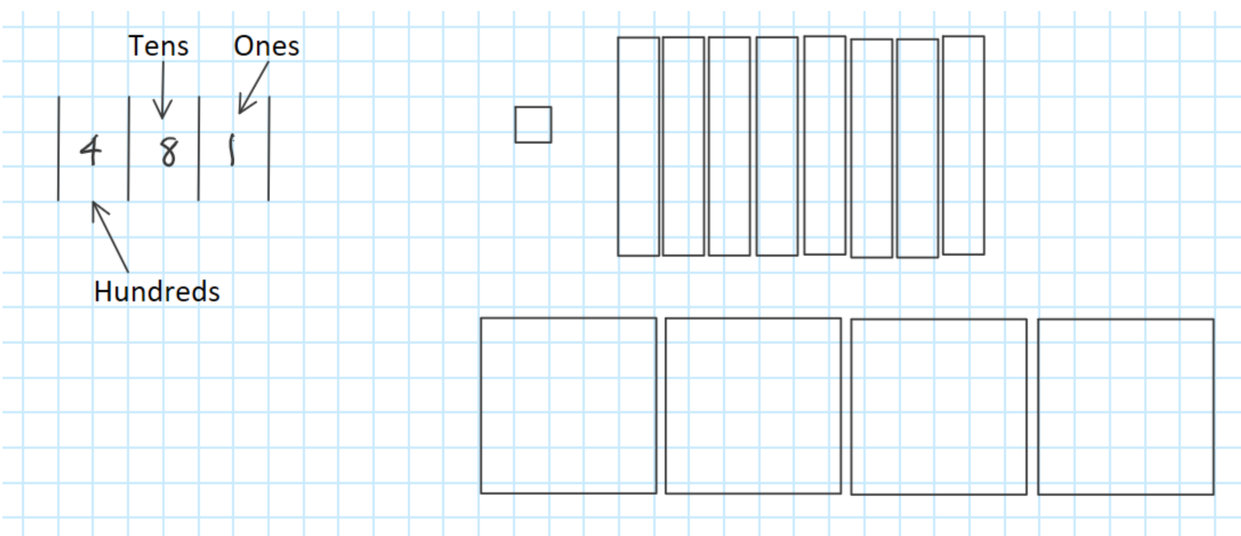
Place Value, Decimals, and Scientific Notation

Place Value

How many 1000s are in one million?

This should not be too difficult a problem to solve. When we look at place values, the answer should be straight forward. There are a thousand groups of one thousand in one million. Similarly, there are ten tens in a hundred. This is how kindergarteners learn about numbers, and the difference between 1 and 10. In kindergarten, children learn to count by 10s. In first grade, they count by 100s. When they get to ten 1000s, many children say that they have ten hundreds instead of one thousand. And this is not technically wrong; there are ten groups of 100 in 1000.

The place values are a reflection over the one column, with the left going off to the tens, thousands, ten thousands, and the right going off to the tenths, thousandths, and ten thousandths. This is shown below. Children learn about how each of these columns works with blocks. Children are given individual blocks and told that ten blocks can be put into one stack of ten blocks. They are taught that ten of these stacks can be turned into a plate, with ten stacks of ten blocks. And once they have ten plates, they can turn those ten plates into a large cube of ten plates of ten stacks of ten cubes.



As seen above, stacks, plates and cubes are shown to be associated with different columns in a number. Stacks are the tens column, plates are the hundreds column, and large cubes are the thousands column. And children are taught different ways to express the same value. 2481 can be written as 2000+400+80+1, which is called the expanded form of a number. We can also express it in terms of powers, or 2*103+4*102+8*101+1*100.

This concept, also, correlates with geometry. Our individual blocks can be viewed as a point, which has 0 dimensions. If we drag our point into a line, or build our individual blocks into a stack, we get something with one dimension. If we drag that line into a plane, or build our stacks into a plate, we get 2 dimensions. We can keep going further into 3 dimensions, etc. This relates the equation above as our one's place (point) is multiplied by 100 (0 dimensions), our tens place (line) is multiplied by 101 (1 dimension), our hundred place (plane) is multiplied by 102 (2 dimensions), etc.

Looking at 1000, we see we can write it as 103= 10*10*10. If we divide this value by 10, we get 10*10*1010=10*10=102=100. We can also divide 1 by 10, which will give us 0.1. But what does .1 really mean? It means that we have one tenth of our unit of measurement. One hundred of something, is relative to your unit of measurement. A fraction of something is a portion of it relative to one whole. When we look at fractions, we need to change our unit from a solid cube to something we can break into smaller portions, like a candy bar. However, changing the types of units that we are measuring, from cubes to plates, for instance, can be really confusing for young children. It is very important to always tell students what the whole we are looking for is - is our unit one cube, one plate, or one candy bar? Let us say our unit is one large cube. When we look at the number 1.312, kids learn that this number is a thousandth, with 312 pieces of the whole unit. In this case we would have 1 large cube, with 312 thousandth pieces of the large cube, or 312 small cubes.

Repeating Decimals

How can we tell whether or not a decimal will repeat forever? Since we use the decimal number system, base 10, we find that a number must be able to divide into pieces of ten. Two and five can divide 10, 4 can divide 100, which is a type of ten, and so on. Numbers that will never divide 10 create a repeating decimal - for example 3, 6, 7, and 9. We see this with ⅓ - because 3 will not divide into 10 to any power.

Think about a piece of string. If we have the natural numbers written on it, how do we create halves? Well, we can just fold the string into halves. However, we cannot fold a string to get $\sqrt{2}$. In third grade, children learn that ¾ is simply 3 groups of 1/4. Children are taught absolute counting, the idea that everything has an absolute value. When asked how much bigger 12 is than 4, children will respond that it is 12 is 8 bigger than 4.

Scientific Notation

When numbers are written in scientific notation, they are written as an integer plus a decimal, multiplied by some power of 10. How do we multiply two numbers written in decimal notation?

Consider the problem (8.31*103)(2.1*105). We can multiply 2.1 and 8.31 to get 17.451. We can convert this to scientific notation by pulling out 10, or 1.7451*10. Then, we can multiply 103 and 105 together, along with our 10 from 1.7451. Then we end up with 1.7451*109.

Definitions

What are the place values?

- The value represented by a digit in a number on the basis of its position inside of the number
- For example, in the number 1,234,567.891
    - The 1 represents the millions; 2 the hundred thousands; 3 ten thousands; 4 thousands; 5 hundreds; 6 tens; and 7 ones
    - When looking at the decimals we have 8 tenths; 9 hundredths; and 1 thousandths
- Place values after the decimal point are denoted with the suffix “ths”

Decimal Definition

- A decimal is any numerical value to the right of a decimal point, system of numbers based on the number 10 (tenth parts)

Scientific Notation Definition

- A method for expressing a given quantity as a number having significant digits necessary for a specified degree of accuracy. Multiplied by 10 to the appropriate power.

Curriculum:

Students start seeing and manipulating numbers with scientific notation in and around 8th Grade. This is where students use numbers expressed in the form of a single digit multiplied by an integer power of 10. The student, when comfortable, also starts performing operations in scientific notation. STANDARDS 8.EE.3 and 8.EE.4