Math 2200-002/Discrete Mathematics
Midterm 2 Review

Enhanced Euclid, Induction and Well-Ordering, Counting.

Enhanced Euclid. Be able to implement Euclid's algorithm with enhancements and use it to solve congruences of the form $mx \equiv b \pmod{n}$.

Sample Problems. Solve the following:

- Find multiplicative inverses of each number from 1 to 30 (mod 31).
- Solve the equation $30a + 42b = 6$.
- Find all 6 solutions (between 1 and 41) to the equation:

$$30x \equiv 18 \pmod{42}$$

Induction. Be able to state the principles of basic induction, strong induction and the well-ordered axiom and to use them in proofs. They are logically equivalent, but you will not be asked to prove this.

Sample Problems. Prove the following.

- For all $r \neq 1$, $r + r^2 + \cdots + r^n = \frac{r^{n+1}-1}{r-1} - 1$.
- For all $r \in \mathbb{R}$, $r \cdot r^2 \cdot \cdots \cdot r^n = r^{\frac{(n+1)n}{2}}$
- Every natural number $n > 1$ is a product of finitely many primes.
- The division algorithm (state and prove it).
- Given $m, n \in \mathbb{N}$, there are integers a, b so that $am+bn = \gcd(m, n)$. (Use well-ordering to prove this, as in your homework assignment)

Counting. Know the basic counting principles and the pigeon-hole principle(s), permutations, combinations and the binomial theorem.

Sample Problems.

- How many 4 digit palindrome numbers are there?
- How many words are there consisting of 3 letters with no repeats?
- How many numbers between 1 and 1000 are not divisible by 2 or 3? How many are not divisible by 2, 3 or 5?
- State the pigeonhole principle and its generalization.
- Prove that in every subset of $\{1, 2, ..., 10\}$ with 6 elements, one of the elements divides another.
- Expand $(x + y)^7$.
- Find the number of possible committees of 4 out of a group of 8.
- What are $P(n, r)$ and $C(n, r)$ and how are they related?