Math 2200-002/Discrete Mathematics
Midterm 1 Review

Logic, Quantifiers, Sets, Functions, Sequences and Number Theory.

Logic. The basic operations are $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ (I don't care about $\oplus$).
Basic definitions include: logical proposition, tautology, contradiction.

You need to be able to:

- Make truth tables for compound logical propositions.
- Check the laws for the arithmetic of logical operators.
- Negate a logical proposition using DeMorgan's Laws.
- Convert English to logical propositions and vice versa.
- Prove logical propositions.

Quantifiers. These are $\exists$ and $\forall$ (and $\exists!$). They can be **nested**.
A quantified proposition has a propositional function and a domain.

You need to be able to:

- Negate a quantified expression using DeMorgan's laws.
- Negate a nested quantified expression using DeMorgan's laws.
- Convert English to quantified expressions and vice versa.
- Prove quantified statements.

Sets. The basic operations are: $\cap, \cup, -, \ ^c, \times$.
Basic concepts: $\emptyset$, $\in$, $\subset$ universe, power set, truth sets.

You need to be able to:

- Take complements of unions and intersections (DeMorgan's Laws).
- Define the operations on sets using logic and quantifiers.
- Prove some set identities.

Functions. Graphs, domain, codomain, range.
Basic concepts: Injective (one-to-one), surjective (onto), bijective.

You need to be able to:

- Find the range of a function.
- Use logic and quantifiers to define injective, surjective, bijective.
- Check whether a function is injective, surjective, bijective
- Explain what a *relation* is.

Sequences. A sequence is $f : \mathbb{N} \rightarrow S$, written $\{a_n\}$ (where $a_n = f(n)$).
Basic concepts: Arithmetic, geometric, Fibonacci, recurrence relation, polynomial and exponential growth, the sequence of partial sums.

You need to be able to:

- Sum some basic sequences.
- Find the nth term of a sequence with a recurrence relation.

Number Theory. Basic operations: **mod** and **div**, $+_n$ and $*_n$.
Basic concepts: Divisibility, division algorithm, congruence (mod m), primes, relatively prime, prime factorizations, gcd, lcm, sieve of Eratosthenes, Euclidean algorithm, Bézout's identity.

You need to be able to:

- Prove basic divisibility Theorems.
- Do some modular arithmetic.
- Run Euclid's algorithm.
- Prove the uniqueness of prime factorizations.